Stat 512 — Take home exam V (due on Aug 6th)

1. Seasonal ranges (in hectares) for alligators were monitored on a lake outside Gainesville, Florida, by biologists from the Florida Game and Fish Commoission. Five alligators monitored in the spring showed ranges of 7.8, 12.3, 8.3, 18.4 and 31. Four different alligators monitored in the summer showed ranges of 102.3, 81, 55.2 and 51. Estimate the difference between mean spring summer ranges, with a 95% confidence interval. (10 pts) Hint: you can use the following code in R to get S_1^2 and S_2^2: (10 pts)

```
data1<-c(7.8, 12.3, 8.3, 18.4,31)
#######S1-square#########
var(data1)
data2<-c(102.3, 81, 55.2, 51)
#######S2-square#########
var(data2)
```

2. Suppose $Y_1, \ldots, Y_n \sim$ Poisson(λ).

   a. Show that $\bar{Y}$ is an efficient estimator of λ using CRLB. (5 pts)

   b. Find the MLE for λ. (5 pts)

3. Suppose $Y_1, \ldots, Y_n$ is a random sample from the pdf

$$f(y|\theta) = \theta y^{-2}, \qquad 0 < \theta \leq y < \infty$$

   a. What is a sufficient statistic for θ? Hint: use Factorization Theorem. (5 pts)

   b. Use Maximum Likelihood Method to obtain an estimator for θ, denoted is as $\hat{\theta}$. (5 pts)

   c. Use moment method to obtain an estimator for θ, denoted is as $\widetilde{\theta}$. Hint: It is possible that it does not exist. (5 pts)

4. Let $Y_1, \ldots, Y_n$ be i.i.d with pdf:

$$f(y|\theta) = \theta y^{\theta-1}, \qquad , 0 \leq y \leq 1, \quad , 0 < \theta < \infty$$

   a. Prove the MLE of θ is $\hat{\theta} = \dfrac{n}{\sum -log(y_i)}$. (10 pts)

b. Is the MLE in part (a) biased? Hint: Find out the distribution of $-log(y_i)$ first. (10 pts)

c. Find the method of moments estimator for θ. (5 pts)

d. Find a complete sufficient statistic for θ. (10 pts)

e. Find the MVUE for θ. (10 pts)

5. Let $Y_1, \ldots, Y_n$ be a random sample from the pdf $f(y|\mu) = e^{-(y-\mu)}$, where $-\infty < \mu < y < \infty$.

a. Show that $Y_{(1)} = min(Y_1, \ldots, Y_n)$ is a complete sufficient statistic. Hint: You will see that $Y_{(1)}$ is not a member of exponential family. So first prove it is sufficient, then prove it is complete. (10 pts)

b. Find the MVUE for μ. (10 pts)